Philadelphia Area Number Theory Seminar

Karen Taylor
Bronx Community College, CUNY and Bryn Mawr College

Quadratic Identities and Maass Waveforms

Abstract: Andrews, Dyson, and Hickerson (ADH) studied the Fourier coefficients of the function

$$\sigma(q) = 1 + \sum_{n=1}^{\infty} \frac{q^{n(n+1)/2}}{(1+q)(1+q^2)\cdots(1+q^n)},$$

where σ is a function that appears in the work of Ramanujan. They prove, among other things, that, for $m > 0$ and $m \equiv 1 \bmod 24$, that these Fourier coefficients are given by

$$T(m) = \#\left\{\begin{matrix}\text{equivalence classes } [(x,y)] \text{ of solutions to} \\ x^2 - 6y^2 = m \text{ with } x + 3y \equiv 1 \bmod 12\end{matrix}\right\} - \#\left\{\begin{matrix}\text{equivalence classes } [(x,y)] \text{ of solutions to} \\ x^2 - 6y^2 = m \text{ with } x + 3y \equiv 5 \bmod 12\end{matrix}\right\}.$$

Cohen showed that

$$\varphi_0(\tau) = y^{1/2} \sum_{\substack{n \in \mathbb{Z} \\ n \neq 0}} T(n) e^{2\pi i n x/24} K_0\left(\frac{2\pi|n|y}{24}\right)$$

is a Maass waveform on $\Gamma_0(2)$. Zweger was able to place $\varphi_0(\tau)$ in a larger framework of indefinite theta functions.

In this talk, I will discuss the problem of placing quadratic identities arising in the work of ADH into a modular framework. This is joint work, in progress, with Larry Rolen.

Wednesday, February 14, 2018, 2:40 – 4:00 PM

Bryn Mawr College, Department of Mathematics
Park Science Center 328 Tea and refreshments at 2:20PM in Park 339